

## Note on the Formal Solution of the Tomonaga-Schwinger Equation.

Smio TANI.

*Department of Physics, Tokyo University.*

(Received Feb. 5, 1949)

We shall present here some remarks on the formal solution of the Tomonaga-Schwinger equation of field theory. However, the physical interpretation of these formal solutions will be left for future works which will come into contact with the fundamental difficulties underlying our present theory.

The fundamental equation of the Tomonaga-Schwinger theory<sup>(1)(2)</sup> is of the form

$$\left\{ H(x) - i \frac{\delta}{\delta \sigma(x)} \right\} \Psi[\sigma] = 0, \quad (1)$$

where the interaction density  $H(x)$  is subject to the condition

$$[H(x), H(x')] = 0, \quad \text{when } (x_\mu - x'_\mu)^2 > 0. \quad (2)$$

The formal solution of this equation will be given by the following state functional

$$\Psi[\sigma] = S[\sigma, \sigma_0] \Psi[\sigma_0], \quad (3)$$

when we impose an initial condition on the hyper-surface  $\sigma_0$  and  $\Psi[\sigma_0]$  denotes the state functional corresponding to this initial condition. The transformation functional  $S[\sigma, \sigma_0]$  is to be determined by the following functional differential equation

$$i \frac{\delta S[\sigma, \sigma_0]}{\delta \sigma(x)} = H(x) S[\sigma, \sigma_0] \quad (4)$$

and it should also have the property

$$S[\sigma_0, \sigma_0] = 1 \quad (5)$$

that is, in form of a functional integral equation, it is given by

$$S[\sigma, \sigma_0] = 1 + \frac{1}{i} \int_{\sigma_0}^{\sigma} H(x') S[\sigma', \sigma_0] dx'. \quad (6)$$

We shall expand the transformation functional  $S[\sigma, \sigma_0]$  into a power series of the coupling constant  $\epsilon$  for the later convenience. (We may assume the interaction density to be of the first order in  $\epsilon$  without missing the essential feature of our argument.)

$$S[\sigma, \sigma_0] = \sum_{n=0}^{\infty} \overset{(n)}{S}[\sigma, \sigma_0]; \quad \text{with} \quad \overset{(0)}{S}[\sigma, \sigma_0] = 1. \quad (7)$$

The functional differential equation for the  $\overset{(n)}{S}[\sigma, \sigma_0]$  is read as

$$i \frac{\delta \overset{(n)}{S}[\sigma, \sigma_0]}{\delta \sigma(x)} = H(x) \overset{(n-1)}{S}[\sigma, \sigma_0] \quad (8)$$

which has the formal solution

$$\overset{(n)}{S}[\sigma, \sigma_0] = \frac{1}{i} \int_{\sigma_0}^{\sigma} H(x') \overset{(n-1)}{S}[\sigma', \sigma_0] dx'. \quad (9)$$

We also have the following equations concerning to their Hermitian conjugate  $\overset{(n)}{S}^+[\sigma, \sigma_0]$

$$-i \frac{\delta \overset{(n)}{S}^+[\sigma, \sigma_0]}{\delta \sigma(x)} = \overset{(n-1)}{S}^+[\sigma, \sigma_0] H(x), \quad (8')$$

$$\overset{(n)}{S}^+[\sigma, \sigma_0] = i \int_{\sigma_0}^{\sigma} \overset{(n-1)}{S}^+[\sigma', \sigma_0] H(x') dx', \quad (9')$$

$$\text{with} \quad \overset{(0)}{S}^+[\sigma, \sigma_0] = 1. \quad (7')$$

1°) We show explicitly that  $S[\sigma, \sigma_0]$  is an unitary operator:

$$\begin{aligned} S^+[\sigma, \sigma_0] S[\sigma, \sigma_0] &= 1 + (\overset{(1)}{S}^+ + \overset{(1)}{S}) + (\overset{(2)}{S}^+ + \overset{(1)}{S}^+ \overset{(1)}{S} + \overset{(1)}{S}) \\ &\quad + (\overset{(3)}{S}^+ + \overset{(2)}{S}^+ \overset{(1)}{S} + \overset{(1)}{S}^+ \overset{(2)}{S} + \overset{(1)}{S} \overset{(2)}{S}) + \dots \\ &= 1. \end{aligned} \quad (10)$$

The proof for the  $\epsilon$ -part of the above equation is self-evident;

$$\overset{(1)}{S}^+ + \overset{(1)}{S} = i \int_{\sigma_0}^{\sigma} H(x') dx' + \left(-\frac{1}{i}\right) \int_{\sigma_0}^{\sigma} H(x') dx' = 0.$$

For the  $\epsilon^2$ -part we require the following relation

$$\begin{aligned} \overset{(2)}{S}^+ + \overset{(1)}{S}^+ \overset{(1)}{S} + \overset{(1)}{S} \overset{(2)}{S} &= (i)^2 \int_{\sigma_0}^{\sigma'} H(x'') dx'' \int_{\sigma_0}^{\sigma} H(x') dx' \\ &\quad + \int_{\sigma_0}^{\sigma} H(x') dx' \int_{\sigma_0}^{\sigma'} H(x'') dx'' + \left(-\frac{1}{i}\right)^2 \int_{\sigma_0}^{\sigma} H(x') dx' \int_{\sigma_0}^{\sigma'} H(x'') dx'' = 0 \end{aligned}$$

wherein the point  $x'$  lies on the surface  $\sigma'$ . We change the notation in the first integral and have the following three integrals

$$\left[ - \int_{\sigma_0}^{\sigma} dx'' \int_{\sigma_0}^{\sigma''} dx' + \int_{\sigma_0}^{\sigma} dx' \int_{\sigma_0}^{\sigma} dx'' - \int_{\sigma_0}^{\sigma} dx' \int_{\sigma_0}^{\sigma'} dx'' \right] H(x') H(x''),$$

which reduce to

$$\left[ - \int_{\sigma_0}^{\sigma} dx'' \int_{\sigma_0}^{\sigma''} dx' + \int_{\sigma_0}^{\sigma} dx' \int_{\sigma_0}^{\sigma} dx'' \right] H(x') H(x'')$$

In order to see the essential points easier, we shall take the surfaces which are parallel to the spatial coordinate axis. Let  $\sigma_0, \sigma', \sigma, \sigma$  correspond to the surface  $x=t, t', t'', t$  respectively. The surface integral (3-dimensional volume integral) of the interaction density over the surface with the time value  $t$  is denoted by

$$\bar{H}(t) = \int_{\sigma} H(x) dv; \quad x_0=t \text{ on the surface } \sigma.$$

Then we shall have

$$\left[ - \int_{t_0}^t dt'' \int_{t_0}^{t''} dt' + \int_{t_0}^t dt' \int_{t_0}^t dt'' \right] \bar{H}(t') \bar{H}(t'')$$

Both integral cover just the same region, though the order of the integration is reversed; so they cancel each other.

The proof for the 3-rd or higher order is a little more complicated, but can be performed similarly by considering the order of the integration in a certain multiple integral.

2°) One can easily verify that there exists the Heisenberg picture of the energy operator in the generalized sense, that is, it is defined in terms of the coordinate of the respective point alone but does not depend on the surface  $\sigma$ : the representative state functional is thereby the same and constant for all world points.

For the proof we have only to show that the transformed operator  $iS^+[\sigma, \sigma_0] \cdot \partial/\partial\sigma(x) \cdot S[\sigma, \sigma_0] = S^+[\sigma, \sigma_0] H(x) S[\sigma, \sigma_0]$  is independent on the form of the space like surface  $\sigma$ , or in other words, their functional derivative with respect to another point  $x$  which lies upon the surface  $\sigma$  always vanishes.

$$\begin{aligned} & \frac{\partial(S^+[\sigma, \sigma_0] H(x) S[\sigma, \sigma_0])}{\partial\sigma(x')} \\ &= \frac{\partial S^+[\sigma, \sigma_0]}{\partial\sigma(x')} H(x) S[\sigma, \sigma_0] + S^+[\sigma, \sigma_0] H(x) \frac{\partial S[\sigma, \sigma_0]}{\partial\sigma(x')} \end{aligned}$$

$$= \frac{1}{i} S^+[\sigma, \sigma_0] (H(x') H(x) - H(x) H(x')) S[\sigma, \sigma_0] = 0. \quad (11)$$

We shall employ the following notation hereafter

$$S^+[\sigma, \sigma_0] H(x) S[\sigma, \sigma_0] = \sum_{n=1}^{\infty} \overset{(n)}{E}(x); \quad \text{with } \overset{(1)}{E}(x) \equiv H(x). \quad (12)$$

(Rigorously speaking, they ought to be written as  $\overset{(n)}{E}_{[\sigma_0]}(x)$ ; See Art. 5.)

The statement that the transformed operator of energy is a point function is valid not only for the transformed operator as a whole, but also for the individual part of the power series of the coupling constant.

3°) The condition for an arbitrary operator  $O(x)$  to have the Heisenberg picture in the sense of the preceding article is

$$[H(x), O(x')] = 0, \quad \text{when } (x_\mu x'_\mu)^2 > 0 \quad (13)$$

The proof is quite similar to that of the preceding article.

4°) The transformation functional  $S[\sigma, \sigma_0]$  can be expressed in terms of the 4-dimensional volume integrals of the point function operators of energy mentioned in Art. 2.: the upper and lower limit of these integrals are bounded by the surfaces  $\sigma$  and  $\sigma_0$  respectively. That is, the  $\overset{(n)}{S}[\sigma, \sigma_0]$ 's are expressed in terms of  $\int_{\sigma_0}^{\sigma} \overset{(n)}{E}(x') dx'$ 's by solving the following equations inversely.

$$\begin{aligned} \frac{1}{i} \int_{\sigma_0}^{\sigma} \overset{(n)}{E}(x') dx' &= \sum_{m=1}^n m \overset{(n-m)}{S}^+[\sigma, \sigma_0] \overset{(m)}{S}[\sigma, \sigma_0] \\ &= n \overset{(n)}{S}[\sigma, \sigma_0] - \sum_{m=1}^{n-1} m \overset{(n-m)}{S}[\sigma, \sigma_0] \overset{(m)}{S}[\sigma, \sigma_0] + \sum_{m=1}^{n-2} \sum_{\substack{\text{All permutation} \\ (i) + (ii) = n-m}} m \overset{(i)}{S} \overset{(ii)}{S} \overset{(m)}{S} \\ &+ \dots + (-1)^l \sum_{m=1}^{n-l} \sum_{\substack{\text{All perm.} \\ (i) + (ii) + \dots + (\lambda) = n-m}} m \overset{(i)}{S} \overset{(ii)}{S} \dots \overset{(\lambda)}{S} \overset{(m)}{S} + \dots + (-1)^{n-1} \{ \overset{(1)}{S} \}^n. \end{aligned} \quad (14)$$

Proof: we notice first that

$$\overset{(n)}{E}(x) = \sum_{m=0}^{n-1} S^+[\sigma, \sigma_0] H(x) \overset{(n-m-1)}{S}[\sigma, \sigma_0] \quad (12')$$

and that

$$\sum_{m=0}^n S^+[\sigma, \sigma_0] \overset{(m)}{S}[\sigma, \sigma_0] = 0, \quad (10)$$

which relation was used in the third statement of Eq (14), i.e.

$$S^{(n)}[\sigma, \sigma_0] = -S^{(n)}[\sigma, \sigma_0] + \sum_{m=1}^{n-1} S^{(m)}[\sigma, \sigma_0] S^{(n-m)}[\sigma, \sigma_0] + \dots + (-1)^l \sum_{\substack{\text{All perm.} \\ (i)+(ii)+\dots+(\lambda)=n}}^{(i)(ii)\dots(\lambda)} S^{(i)} S^{(ii)} \dots S^{(\lambda)} + \dots + (-1)^n \{S^{(1)}[\sigma, \sigma_0]\}^n \quad (15)$$

By integrating the following equation

$$S^{(n)}[\sigma, \sigma_0] H(x) S^{(m)}[\sigma, \sigma_0] = i \frac{\delta}{\delta \sigma(x)} \left\{ \sum_{l=0}^n S^{(n-l)}[\sigma, \sigma_0] S^{(m+1+l)}[\sigma, \sigma_0] \right\}, \quad (16)$$

we have the relation

$$\frac{1}{i} \int_{\sigma_0}^{\sigma} S^{(n)}[\sigma', \sigma_0] H(x') S^{(m)}[\sigma', \sigma_0] dx' = \sum_{l=0}^n S^{(n-l)}[\sigma, \sigma_0] S^{(m+1+l)}[\sigma, \sigma_0]. \quad (17)$$

Then, from Eqs. (17) and (12') we get the final results.

The results for the first four order in  $\epsilon$  are tabulated as follows :

$$\begin{aligned} S^{(1)}[\sigma, \sigma_0] &= \frac{1}{i} \int_{\sigma_0}^{\sigma} E^{(1)}(x') dx'; \\ S^{(2)}[\sigma, \sigma_0] &= \frac{1}{2!} \left\{ \frac{1}{i} \int_{\sigma_0}^{\sigma} E^{(1)}(x') dx' \right\}^2 + \frac{1}{2} \frac{1}{i} \int_{\sigma_0}^{\sigma} E^{(2)}(x') dx; \\ S^{(3)}[\sigma, \sigma_0] &= \frac{1}{3!} \left\{ \frac{1}{i} \int_{\sigma_0}^{\sigma} E^{(1)}(x') dx' \right\}^3 + \frac{1}{3!} \left( \frac{1}{i} \right)^2 \int_{\sigma_0}^{\sigma} E^{(2)}(x') dx' \int_{\sigma_0}^{\sigma} E^{(1)}(x') dx' \\ &\quad + \frac{1}{3} \left( \frac{1}{i} \right)^2 \int_{\sigma_0}^{\sigma} E^{(1)}(x') dx' \int_{\sigma_0}^{\sigma} E^{(2)}(x') dx' + \frac{1}{3} \frac{1}{i} \int_{\sigma_0}^{\sigma} E^{(3)}(x') dx'; \\ S^{(4)}[\sigma, \sigma_0] &= \frac{1}{4!} \left\{ \frac{1}{i} \int_{\sigma_0}^{\sigma} E^{(1)}(x') dx' \right\}^4 + \frac{1}{4!} \left( \frac{1}{i} \right)^3 \int_{\sigma_0}^{\sigma} E^{(2)}(x') dx' \left\{ \int_{\sigma_0}^{\sigma} E^{(1)}(x') dx' \right\}^2 \\ &\quad + \frac{1}{4 \cdot 2} \left( \frac{1}{i} \right)^2 \left\{ \int_{\sigma_0}^{\sigma} E^{(2)}(x') dx' \right\}^2 + \frac{1}{4 \cdot 2} \left( \frac{1}{i} \right)^3 \left\{ \int_{\sigma_0}^{\sigma} E^{(1)}(x') dx' \right\}^2 \int_{\sigma_0}^{\sigma} E^{(2)}(x') dx' \\ &\quad + \frac{1}{4 \cdot 3} \left( \frac{1}{i} \right)^2 \int_{\sigma_0}^{\sigma} E^{(3)}(x') dx' \int_{\sigma_0}^{\sigma} E^{(1)}(x) dx' \\ &\quad + \frac{1}{4 \cdot 3} \left( \frac{1}{i} \right)^3 \int_{\sigma_0}^{\sigma} E^{(1)}(x') dx' \int_{\sigma_0}^{\sigma} E^{(2)}(x') dx' \int_{\sigma_0}^{\sigma} E^{(1)}(x') dx' \\ &\quad + \frac{1}{4} \left( \frac{1}{i} \right)^2 \int_{\sigma_0}^{\sigma} E^{(1)}(x') dx' \int_{\sigma_0}^{\sigma} E^{(3)}(x') dx' + \frac{1}{4} \frac{1}{i} \int_{\sigma_0}^{\sigma} E^{(4)}(x') dx'. \end{aligned}$$

5°) Between the  $\overset{(n)}{E}(x)$ 's there exists the following recurrence formula

$$(n-1)\overset{(n)}{E}(x) = \sum_{m=1}^{n-1} \left[ \overset{(n-m)}{E}(x), \frac{1}{i} \int_{\sigma}^{\sigma} \overset{(m)}{E}(x') dx' \right], \quad (18)$$

which enables for one to get the higher order terms successively initiating from the  $\overset{(1)}{E}(x) = H(x)$ . This formula is useful in practice, but some points should be noticed as regards to the boundary of the integration region. In the above equation, the integral  $\int_{\sigma}^{\sigma} \overset{(m)}{E}(x') dx'$  should have been written as  $\int_{\sigma_0}^{\sigma} \overset{(m)}{E}(x') dx'$  according to their original meaning, however, the contribution from the lower surface  $\sigma_0$  have nothing to do with the character of  $\overset{(n)}{E}(x)$  as a point function. So it is convenient for one to proceed in the following way: we may neglect the contribution from the lower boundary by employing the Fourier-transform of the integrand and the Gauss' theorem.\* This integral is denoted by  $\int_{\sigma}^{\sigma} \overset{(m)}{E}(x') dx'$  in the above equation. This procedure corresponds physically to the adiabatic switching on of the interaction: and the principal value should be employed in the integrals which contain processes obeying the energy-momentum conservation law. After we get the general point function of energy, we supplement the contribution which come from the surface  $\sigma_0$ , if necessary. The supplementary terms are expressed by means of the surface integrals over  $\sigma_0$  by performing the Fourier-transform of  $\overset{(m)}{E}(x)$ 's.

For the proof of Eq. (18) we should combine the following relations.

$$\overset{(n)}{E}(x) = \sum_{n=0}^n S^+[\sigma, ] \cdot i \frac{\partial}{\partial \sigma(x)} \cdot S[\sigma, ]^{\overset{(n-m)}{}} = i \frac{\partial S[\sigma, ]}{\partial \sigma(x)} - \sum_{m=1}^{n-1} S[\sigma, ]^{\overset{(n-m)}{}} \overset{(m)}{E}(x),^{**}$$

or

$$i \frac{\partial S[\sigma, ]}{\partial \sigma(x)} = \sum_{m=1}^n S[\sigma, ]^{\overset{(n-m)}{}} \overset{(m)}{E}(x), \quad (19)$$

and

---

\* In Schwinger's notation,  $\int_{\sigma}^{\sigma} \overset{(m)}{E}(x') dx' = \int_{\sigma}^{\sigma} G_{\mu}(x') d\sigma_{\mu}'$ , where  $\frac{\partial}{\partial X_{\mu}} G_{\mu}(x) = \overset{(m)}{E}(x)$ , and  $d\sigma_{\mu}'$  is the element at the point  $x'$  of the surface  $\sigma$  over which the integration is performed.

\*\* In  $S[\sigma, ]$ , the  $\int_{\sigma}^{\sigma} \overset{(m)}{E}(x') dx'$ 's are exclusively used,

$$-i \frac{\delta S^+[\sigma, ]}{\delta \sigma(x)} = \sum_{m=1}^n E^{(m)}(x) S^+[\sigma, ], \quad (19')$$

which are revision of (12') by making use of (15); and

$$\frac{1}{i} \int_{\sigma} E^{(n)}(x') dx' = - \sum_{m=1}^n m S^+[\sigma, ] S^{(n-m)}[\sigma, ] = \sum_{m=1}^n m S[\sigma, ] S^{(m)}[\sigma, ], \quad (14')$$

which can be shown similarly as in Eq. (14) by changing the role of the Hermitian conjugate operators. With the aid of Eq. (10), they yield the results

$$\begin{aligned} E^{(n)}(x) &= ni \frac{\delta S[\sigma, ]}{\delta \sigma(x)} + \sum_{m=1}^{n-1} mi \frac{\delta S^+[\sigma, ]}{\delta \sigma(x)} S^{(m)}[\sigma, ] \\ &\quad + \sum_{m=1}^{n-1} (n-m)i S^+[\sigma, ] \frac{\delta S[\sigma, ]}{\delta \sigma(x)} \\ &= nE^{(n)}(x) + n \sum_{m=1}^{n-1} S^{(n-m)}[\sigma, ] E^{(m)}(x) - \sum_{m=1}^{n-1} \sum_{l=1}^{n-m} m E^{(m)}(x) S^+[\sigma, ] S^{(n-m-l)}[\sigma, ] \\ &\quad + \sum_{m=1}^{n-1} \sum_{l=1}^{n-m} (n-m) S^+[\sigma, ] S^{(n-m-l)}[\sigma, ] E^{(l)}(x) \\ &= nE^{(n)}(x) - \sum_{l=1}^{n-1} E^{(l)}(x) \frac{1}{i} \int_{\sigma} E^{(n-l)}(x') dx' + \sum_{l=1}^{n-1} \frac{1}{i} \int_{\sigma} E^{(n-l)}(x') dx' \cdot E^{(l)}(x). \end{aligned}$$

6°) We add that the  $E^{(n)}(x)$  of the preceding article is uniquely determined disregarding an arbitrary constant factor, when we impose a condition that they should be a point function and be a linear combination of the products  $\int_{\sigma} E^{(i)}(x') dx' \int_{\sigma} E^{(ii)}(x'') dx'' \dots H(x) \dots \int_{\sigma} E^{(\lambda)}(x''') dx''' [(i) + (ii) + \dots + (\lambda) = n-1]$  of the integrals of the point function energy lower than  $n$  and  $H(x)$ . We will not reproduce the proof in detail. But the essential is as follows: when we perform the functional differentiation of this linear combination with respect to the point  $x'$  on the surface  $\sigma$ , we have as many condition that the sum of certain coefficients of combination should vanish as there are types of terms such as  $\int_{\sigma} E^{(i)}(x') dx' \int_{\sigma} E^{(ii)}(x'') dx'' \dots E^{(x)}(x') \dots H(x) \dots \int_{\sigma} E^{(\lambda)}(x''') dx''' [(i) + (ii) + \dots + (x) + \dots + (\lambda) = n-1]$ . In general, the number of the conditions is gerater than the number of the unknown coefficients. However, if we eliminate those conditions which are not

linearly independent and take the condition  $[H(x), H(x')] = 0$  into account, there is left just one less conditions than the number of the unknown coefficients.

7°) For some purposes it is convenient to express the transformation functional in form of an exponential function

$$S[\sigma, \sigma_0] = \exp -i(K^{(1)}[\sigma, \sigma_0] + K^{(2)}[\sigma, \sigma_0] + \dots), \quad (20)$$

where the  $K^{(n)}[\sigma, \sigma_0]$  is  $n$ -th order in  $\epsilon$ , and is expressed in terms of  $\int_{\sigma_0}^{\sigma} E^{(m)}(x) dx$ 's. The results for the first four order are as follows:

$$K^{(1)}[\sigma, \sigma_0] = \int_{\sigma_0}^{\sigma} E^{(1)}(x') dx';$$

$$K^{(2)}[\sigma, \sigma_0] = \frac{1}{2} \int_{\sigma_0}^{\sigma} E^{(2)}(x') dx';$$

$$K^{(3)}[\sigma, \sigma_0] = \frac{1}{3} \int_{\sigma_0}^{\sigma} E^{(3)}(x') dx' + \frac{1}{12} \left[ \int_{\sigma_0}^{\sigma} E^{(1)}(x') dx', \int_{\sigma_0}^{\sigma} E^{(2)}(x') dx' \right];$$

$$K^{(4)}[\sigma, \sigma_0] = \frac{1}{4} \int_{\sigma_0}^{\sigma} E^{(4)}(x') dx' + \frac{1}{12} \left[ \int_{\sigma_0}^{\sigma} E^{(1)}(x') dx', \int_{\sigma_0}^{\sigma} E^{(3)}(x') dx' \right].$$

The  $\epsilon^n$ -approximation of the original equation in which the first term of the transformed equation of motion is  $\frac{E^{(n)}(x)}{n}$ , which is a point function, i.e. in equation

$$\left\{ \frac{1}{i} \frac{\delta}{\delta \sigma(x)} + \frac{E^{(n)}(x)}{n} + (\text{terms of order higher than } n \text{ in } \epsilon) \right\} \Phi[\sigma] = 0. \quad (21)$$

is given by the following canonical transformation of the state functional and the dynamical variables

$$\begin{aligned} \Psi[\sigma] &\longrightarrow \Phi[\sigma]; \quad \Psi[\sigma] = U^{(n)}[\sigma, ] \Phi[\sigma] \\ U^{(n)}[\sigma, ] &= \exp -i \left\{ \sum_{m=1}^{n-1} K^{(m)}[\sigma, ] + \left( K^{(n)}[\sigma, ] - \frac{1}{n} \int_{\sigma_0}^{\sigma} E^{(n)}(x') dx' \right) \right\} \\ O(x) &\longrightarrow U^{(n)-1}[\sigma, ] O(x) U^{(n)}[\sigma, ]. \end{aligned} \quad (22)$$

8°) In concluding this note, we can add that the generalization of the results presented here may be able to include the cases where the



interaction density is no more a point function but contains the normal or other quantity related to the surface and consequently the integrability condition is much more complicated than in Eq. (2). But these results will be discussed in another occasion.

The author wishes to express his cordial thanks to Prof. Tomonaga for the suggestion to this work.

**References.**

- (1) S. Tomonaga: *Prog. Theor. Phys.*, 1 (1946) 27, and a series of works following this Part I.
  - (2) J. Schwinger: *Phys. Rev.* 74 (1948) 1439.
-